Prof. Dr. Albin F. Wrulich EPFL, Winter Term 2005/2006

Introduction to Particle Accelerator Physics

Tutorial 5 - Solutions

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1. Apertures, Dispersion, and Acceptance

a) Recall from the lecture the definition of the vertical acceptance (without dispersion)

$$A_y = \min\left(\frac{a_y(s)^2}{\beta_y(s)}\right)$$

where $a_y(s)$ is the vertical acceptance. In this case the vertical acceptance is constant around the ring so the minimum will be reached when the vertical beta function reaches a maximum:

$$A_y = \min\left(\frac{a_y(s)^2}{\beta_y(s)}\right) = \frac{(h/2)^2}{\beta_{y,\max}} = \frac{(15\,\mathrm{mm})^2}{25\,\mathrm{m/rad}} = 9\,\mathrm{mm\,mrad}$$

b) Since we know there is a vertical focus at the center of the straight (symmetry point), we can refer to the result of problem 5 of tutorial 2 for the beta function around a symmetry point:

$$\beta_y(s) = \beta_{y,0} + \frac{s^2}{\beta_{y,0}}$$

From this we gather that the beam will be the largest at the edges of the undulator and therefore this is where we exepct the acceptance limitation:

$$\beta_{y,\mathrm{edge}} = \beta_{y,0} + \frac{(L/2)^2}{\beta_{y,0}} = 1\,\mathrm{m/rad} + \frac{(2\,\mathrm{m})^2}{1\,\mathrm{m/rad}} = 5\,\mathrm{m/rad}$$

The vertical aperture of the undulator is one half of the gap height which leads to

$$A_y = \min\left(\frac{a_y(s)^2}{\beta_y(s)}\right) = \frac{(g_u/2)^2}{\beta_{y,\mathrm{edge}}} = \frac{(3\,\mathrm{mm})^2}{5\,\mathrm{m/rad}} = 1.8\,\mathrm{mm\,mrad}$$

So the installation of the narrow-gap undulator has reduced the vertical acceptance by a factor 5!

c) Remember the definition of the betatron phase advance:

$$\phi(0,l) = \int_0^l \frac{ds}{\beta(s)}$$

This allows us to calculate the total phase advance for the undulator:

$$\phi_{x,y} = \int_{-L/2}^{+L/2} \frac{1}{\beta_{x,y}(s)} ds$$

$$
\begin{aligned}
&= \int_{-L/2}^{+L/2} \frac{1}{\beta_{x,y,0} + \frac{s^2}{\beta_{x,y,0}}} ds \\
&= \beta_{x,y,0} \int_{-L/2}^{+L/2} \frac{1}{\beta_{x,y,0}^2 + s^2} ds \\
&= \left[\arctan \frac{s}{\beta_{x,y,0}} \right]_{s=-L/2}^{s=+L/2} \\
&= 2 \arctan \frac{L/2}{\beta_{x,y,0}} \qquad (1)
\end{aligned}
$$

So we can calculate the total phase advance in x and y:

$$\phi_x = 2 \arctan \frac{L/2}{\beta_{x,0}} = 2 \arctan \frac{2\,\text{m}}{9\,\text{m/rad}} \approx 25^\circ$$

$$\phi_y = 2 \arctan \frac{L/2}{\beta_{y,0}} = 2 \arctan \frac{2\,\text{m}}{1\,\text{m/rad}} \approx 127^\circ$$

d)

$$A_y = \min \left(\frac{a_y(s)^2}{\beta_y(s)} \right) = \min \left(\frac{(g_u/2)^2}{\beta_y(s)} \right) = \frac{(g_u/2)^2}{\hat{\beta}_y}$$

where $\hat{\beta}_y$ is the maximum of the beta function at the edge of the undulator. We can calculate the optimum $\hat{\beta}_{y,0}$ which minimizes the beta function at the undulator edge:

$$\beta_{y,edge} = \beta_{y,0} + \frac{(L/2)^2}{\beta_{y,0}}$$

$$\frac{d\beta_{y,edge}}{d\beta_{y,0}} = 1 - \frac{(L/2)^2}{\beta_{y,0}^2}$$

$$0 \overset{!}{=} \frac{d\beta_{y,edge}}{d\beta_{y,0}} = 1 - \frac{(L/2)^2}{\hat{\beta}_{y,0}^2}$$

Which we then further simplify:

$$\hat{\beta}_{y,0}^2 = (L/2)^2$$

$$\implies \hat{\beta}_{y,0} = \pm L/2$$

For this optimum $\hat{\beta}_{y,0}$ the beta function at the edges of the undulator is minimized:

$$\hat{\beta}_y = \hat{\beta}_{y,0} + \frac{(L/2)^2}{\hat{\beta}_{y,0}} = 2\,\text{m/rad} + \frac{(2\,\text{m})^2}{2\,\text{m/rad}} = 4\,\text{m/rad}$$

And thus the acceptance of the undulator is maximized:

$$\hat{A}_y = \frac{(g_u/2)^2}{\hat{\beta}_y} = \frac{(3\,\text{mm})^2}{4\,\text{m/rad}} = 2.25\,\text{mm mrad}$$

So by properly choosing the parameter $\beta_{y,0}$, we gain 25% in acceptance!

e) Since we require the bending magnets to do the vertical focussing, we can expect the maximum of the vertical beta function $\beta_{y,\max}$ to be inside the bending magnet. We can then take the optimum acceptance $\hat{A}_y$ from above and ask what minimum aperture can be chosen in the bending magnet without limiting the vertical acceptance:

$$\hat{a}_y = \sqrt{\hat{A}_y \cdot \beta_{y,\max}} = \sqrt{2.25\,\mathrm{mm \cdot mrad} \cdot 25\,\mathrm{m}} = 7.5\,\mathrm{mm}$$

So, without limiting the acceptance in the bending magnet the required gap height would be only (2 × 4 mm required for the vacuum chamber thickness):

$$\hat{g}_m = 2 \times 4\,\mathrm{mm} + 2\hat{a}_y = 23\,\mathrm{mm}$$

Compared to the originally planned 38 mm of required magnetic gap height, this is a 40% reduction!

f) Recall Maxwell's equation for the magnetic induction induced by a current running through a coil

$$\oint H \cdot ds = \int\int j \cdot dA$$

Assuming constant current density and evaluating the integral on the LHS

$$I = j \cdot A \propto A \propto g_m$$

From Ohm's law we get the power $P = RI^2$ and if we consider that the ohmic resistance scales inversely with the coil area $R \propto A^{-1}$ we can derive:

$$P = RI^2 \propto A^{-1}A^2 \propto g_m$$

i.e. the power scales linearly with the magnetic gap. Therefore, a 40% reduction of magnetic gap height corresponds to a 40% decrease in power consumption!

2. Chromaticity Correction with Sextupoles in a Collider

No, sextupoles can't be used to correct chromaticity locally in the interaction region. Usually dispersion is suppressed at the IP. However, dispersion is needed to sort particles according to their momentum at the sextupoles. Without dispersion the sextupoles can't correct chromaticity. In addition the very high betatron values around the interaction region would let sextupole magnet errors have devastating influence on the beam.

3. Chromaticity in Linacs

Chromaticity describes the effect of a focussing error of a quadrupole lens for particles which don't have the ideal momentum. Therefor it clearly exists in linacs as well. In order to correct chromaticity in a linac dispersion has to be introduced artificially. This is normally done by using dipole magnets to form a dispersive chicane. The amount of dispersion created is chosen to be as small as possible.

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