

# Introduction to Particle Accelerator Physics

## Tutorial 4 - Problems

Discussion: 17.1.2006

Hand in: 24.1.2006

Solutions: 31.1.2006

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### 1. Quadrupole Errors and Tune Shifts

In the lecture a quadrupole focussing error was introduced into Hill's equation by replacing the focussing function  $K(s)$  with  $K(s) + \Delta k(s)$ . It was then proposed that the error in the focussing function could be represented as a gradient error  $\Delta(kl)$  leading to a tune shift  $\Delta Q = \frac{1}{4\pi} \beta_0 \Delta(kl)$ . Assume that the tune shift is small with respect to the tune and prove this statement.

### 2. Momentum Compaction and Transition Energy

Dispersion leads to path length changes for off-momentum particles. This is characterized by the momentum compaction factor  $\alpha_c$  defined in the lecture. Considering that timing is very important in an accelerator, can path length changes be related to a change of the revolution period? Or in mathematical terms, how does  $\frac{\Delta T}{T}$  depend on  $\frac{\Delta p}{p}$ ? Is it possible to build an accelerator where  $\Delta T = 0$  regardless of  $\Delta p$ ?

### 3. Quadrupole Scan for Emittance Measurement

Assume a screen monitor downstream of a tunable quadrupole magnet. Show how measuring the beam size  $\sigma_x^2$  as a function of quadrupole strength can be used as an emittance measurement.