How do you accurately measure the storage ring energy?

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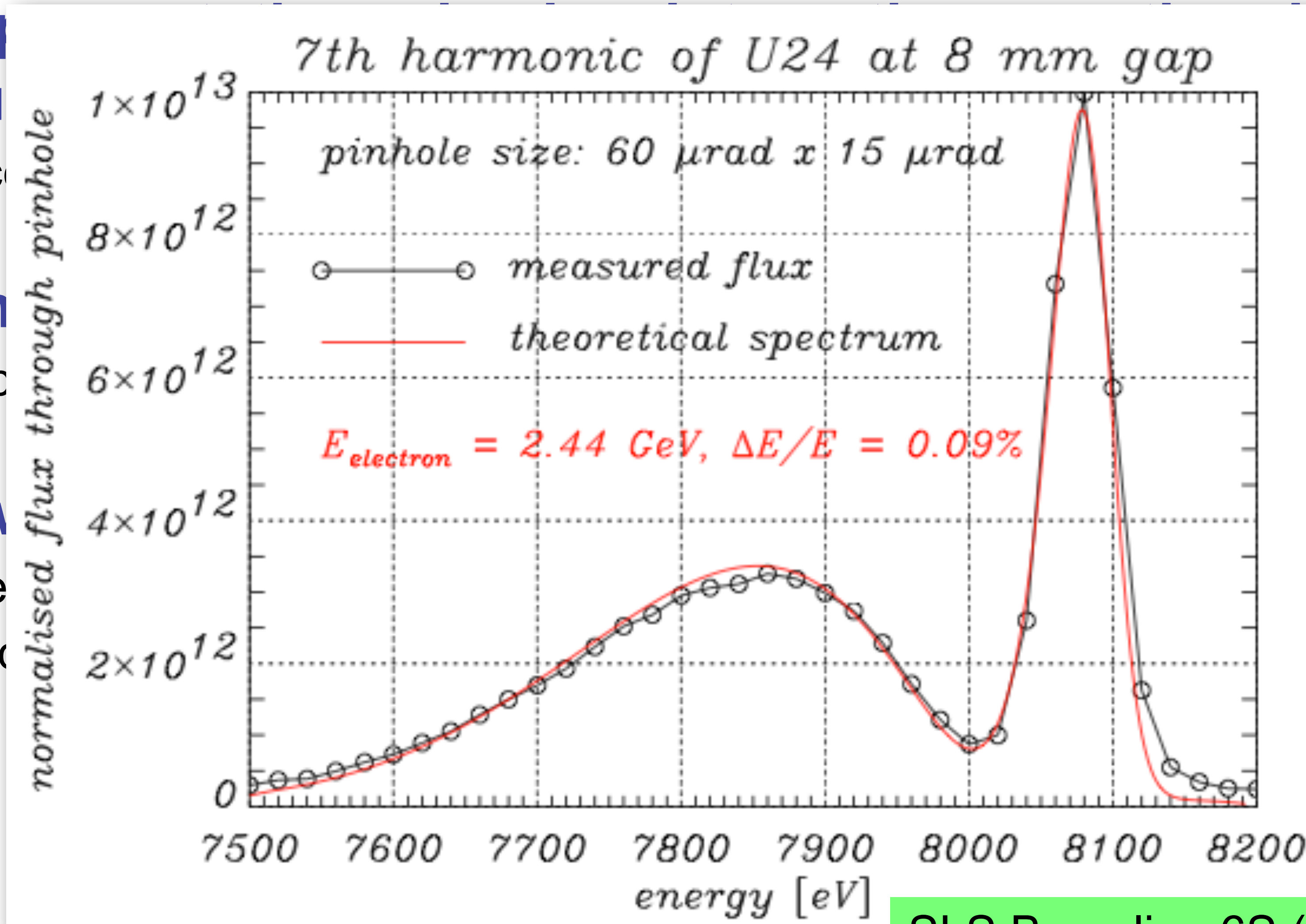

SLS Beamline 6S (PX), 2001

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  - So in the end... who cares at all?

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  - In order to control it, we need to measure it first
  - Determining momentum compaction requires measuring an energy shift as a function of RF detuning

$$-\frac{\Delta f_{\mathrm{rf}}}{f_{\mathrm{rf}}} = \alpha_c \left(\frac{\Delta E}{E}\right) + \alpha_2 \left(\frac{\Delta E}{E}\right)^2 + \ldots$$

Why Resonant Spin Depolarization?

- At MAX-lab we usually don't care too much for short bunches in storage rings, but...

Why Resonant Spin Depolarization?

- At MAX-lab we usually don't care too much for short bunches in storage rings, but...
- When do you ever get a chance to see the quantum nature of anything?

Why R

- At MAX … rt bunc
- When c … n nature of ar

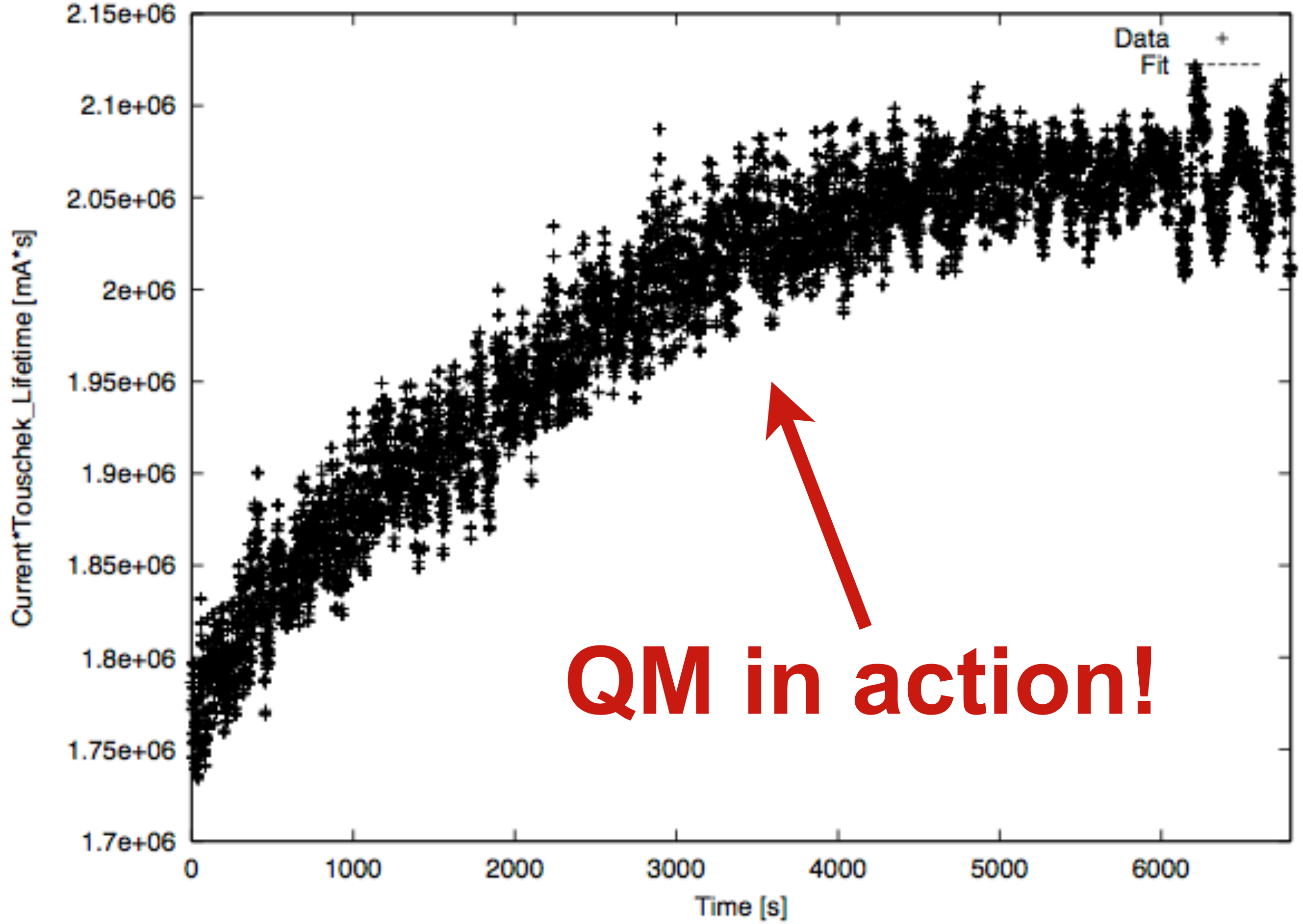

Figure 2: Polarization build-up and fit (flat orbit and vertical BBA). The fit parameter for the characteristic build-up time is (1837 ± 1) s corresponding to an equilibrium polarization of 91%.

SLS, 2001

RSD in a Nutshell

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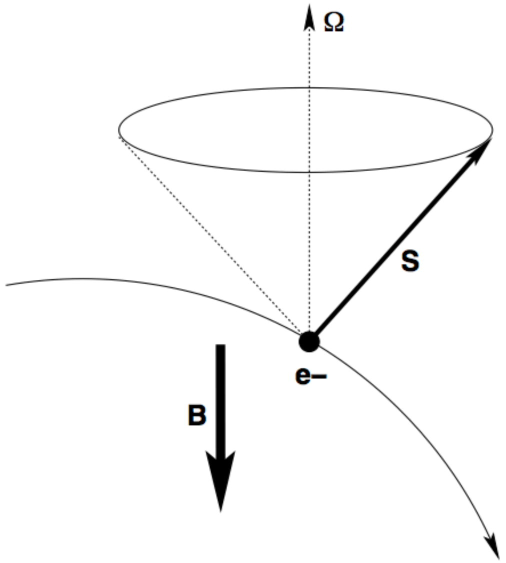

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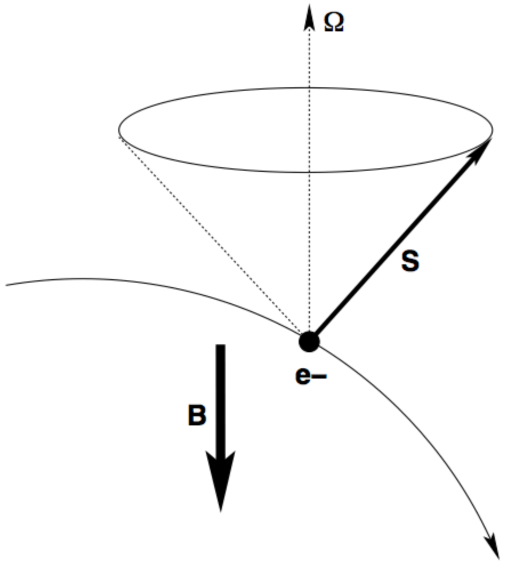

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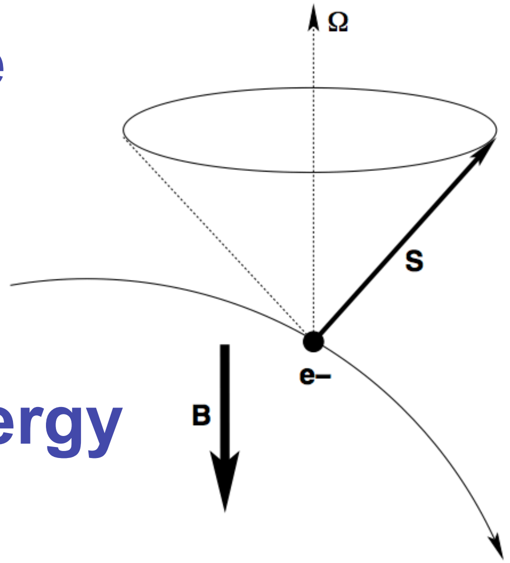
- Detect depolarizing frequency → beam energy



Spin Dynamics in Electron Storage Rings

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$$W_{\uparrow\downarrow} = \frac{5\sqrt{3}}{16} \frac{e^2\gamma^2\hbar}{m_e^2 c^2 \rho^3} \left(1 + \frac{8}{5\sqrt{3}}\right)$$

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$$P_{\mathrm{ST}} = \frac{W_{\uparrow\downarrow} - W_{\downarrow\uparrow}}{W_{\uparrow\downarrow} + W_{\downarrow\uparrow}} = \frac{8}{5\sqrt{3}} = 92.38\%$$

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- **But in a real storage ring things are a bit more complicated... (as always)**

Spin Dynamics in Electron Storage Rings

- **Electron's spin interacts with magnetic fields via the magnetic moment associated with the spin, i.e.**

$$\vec{\mu} = -\frac{ge}{2m_e c}\hbar\vec{S}$$

  – where g is the anomalous magnetic moment of the electron

$$a = \frac{g-2}{2} = 0.00115965218073(28)$$

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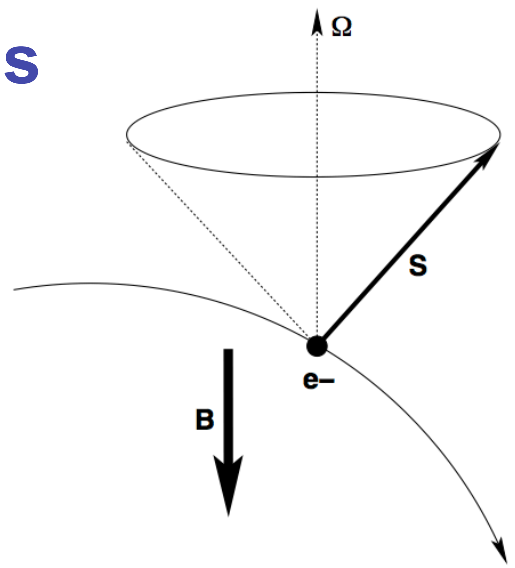
- **In a storage ring the electron spins precess around the guiding dipole field**

$$\frac{d\vec{S}}{dt} = \vec{\Omega} \times \vec{S}$$

  - with the angular velocity (Thomas precession)

$$\vec{\Omega} = -\frac{ge}{2m_e c}\vec{B}$$



Spin Dynamics in Electron Storage Rings

- **Relativistic electrons in lab frame**

$$\vec{\Omega}_{\mathrm{lab}} = -\frac{e}{m_e c}\left[\left(a+\frac{1}{\gamma}\right)\vec{B} - \frac{a\gamma}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B}) - \left(a+\frac{1}{\gamma+1}\right)\vec{\beta}\times\vec{E}\right]$$

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$$\frac{d\vec{S}}{ds} = -\frac{e}{m_e c\gamma}\left[(1+a)\vec{B}_{\parallel} + (1+a\gamma)\vec{B}_{\perp}\right]\times\vec{S}$$

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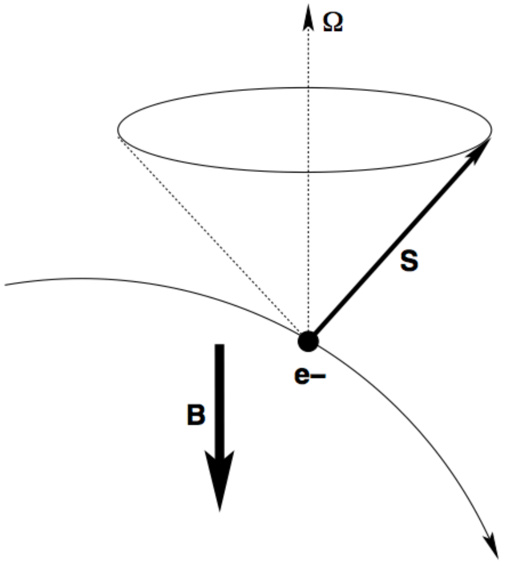

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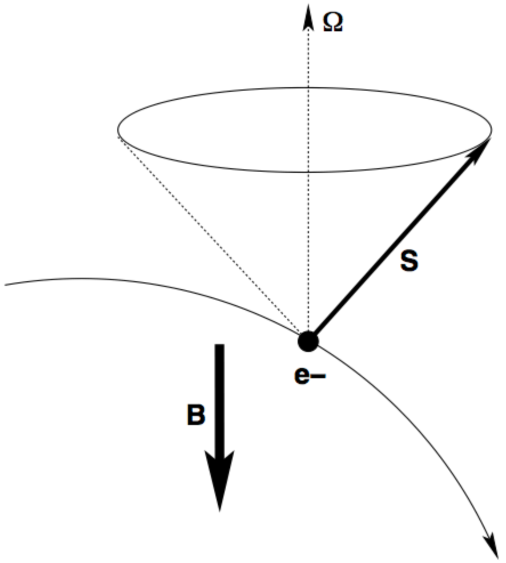
- **But ideal ring has only $B_{\perp}$**

$$\vec{\Omega}_{\rm sp} = \frac{e\vec{B}_{\perp}}{m_e c\gamma}a\gamma$$



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- If we measure this spin tune, we know the energy

Spin Dynamics in Electron Storage Rings

- **But how do we measure the spin precession frequency?**
  - kick spins into machine plane
  - if this is done in resonance with precession, polarization will collapse

Spin Dynamics in Electron Storage Rings

- **But h** ... **ncy?**
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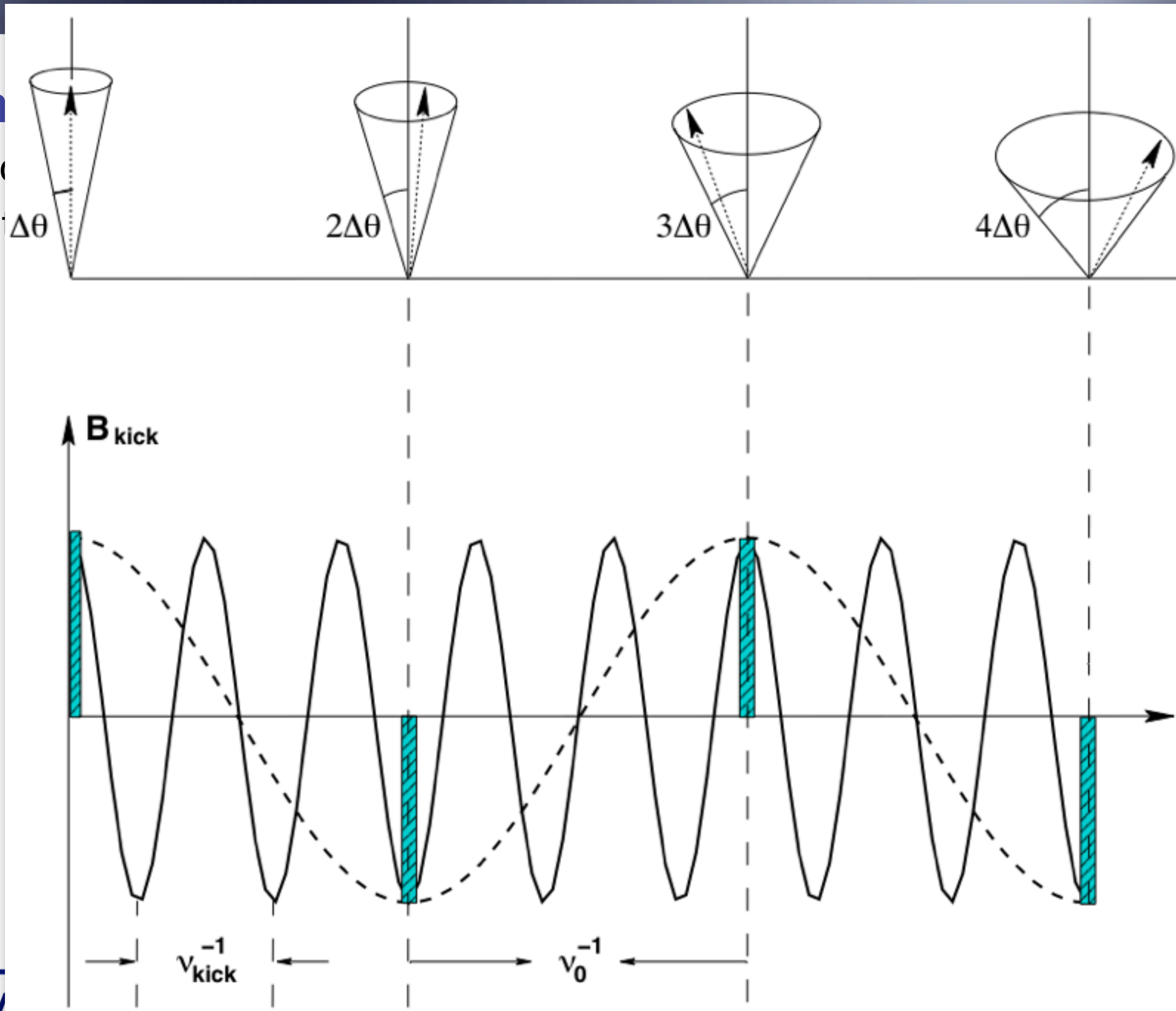

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- **Touschek scattering ➞ Möller scattering cross-section**

$$\text{Möller:}\ \frac{d\sigma_{ts}}{d\Omega} = \frac{4r_e^2}{\beta^4}\left[\frac{4}{\sin^4\theta} - \frac{3+P^2}{\sin^2\theta}\right] \qquad \longrightarrow \qquad \frac{1}{\tau_{ts}} = \langle aC(\xi)\rangle + P^2\langle aF(\xi)\rangle$$

  - if polarization collapses, Touschek scattering cross-section increases

    $\rightarrow$ Touschek lifetime drops
    $\rightarrow$ Touschek losses increase (Touschek losses come in pairs!)

Example: RSD Measurement Campaign

- **Campaign carried out at SLS, 2001-2002**
- **Goals:**
  - calibrate energy with better accuracy than previous Hall probe measurements of dipoles
  - verify if SLS energy was actually 1% too high (as indicated by beam-based quadrupole and sextupole adjustments during commissioning)
  - measure nonlinear momentum compaction and compare to model values

Example: RSD Measurement Campaign

- Step #1: Verify Touschek-dominated beam

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Example: RSD Measurement Campaign

- **Step #1: Verify Touschek-dominated beam**
- **Step #2: Verify high degree of stored beam polarization**

Theory for (perfect) SLS:

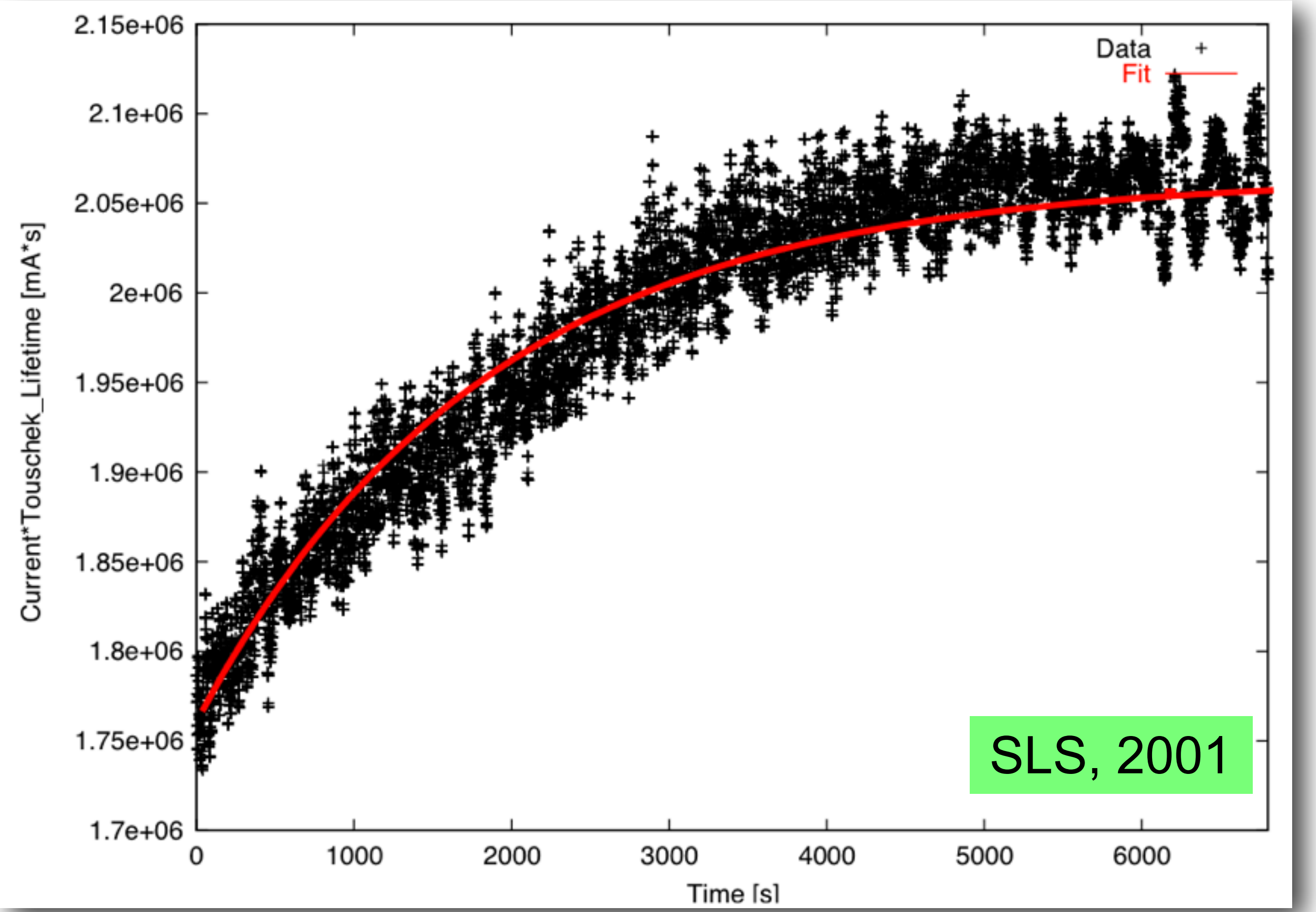
$$\hat{\tau}_p = 1873\text{s}$$

From fit to measured $I\tau$ data:

$$\tau_p = (1837 \pm 1)\ \text{s}$$

Actual polarization level:

$$P_0 = P_{\text{ST}}\frac{\tau_p}{\hat{\tau}_p} = 91\%$$



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- **Step #1: Verify Touschek-dominated beam**
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  - Feed sinusoidal excitation to vertical kicker magnet
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Ex ent Campaign

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- St ed beam polarization
- St onance
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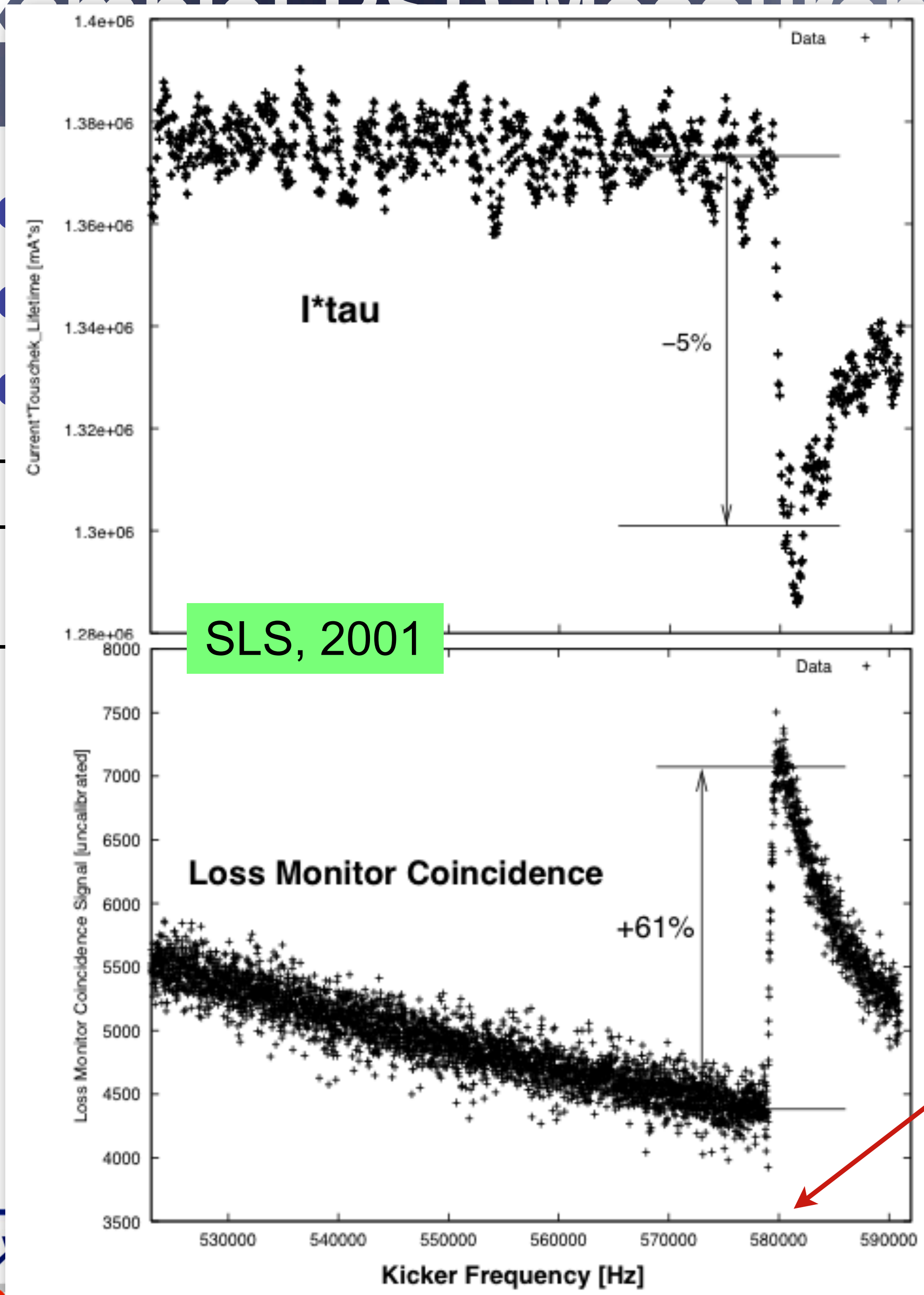

Resonance candidate observed around 580 kHz

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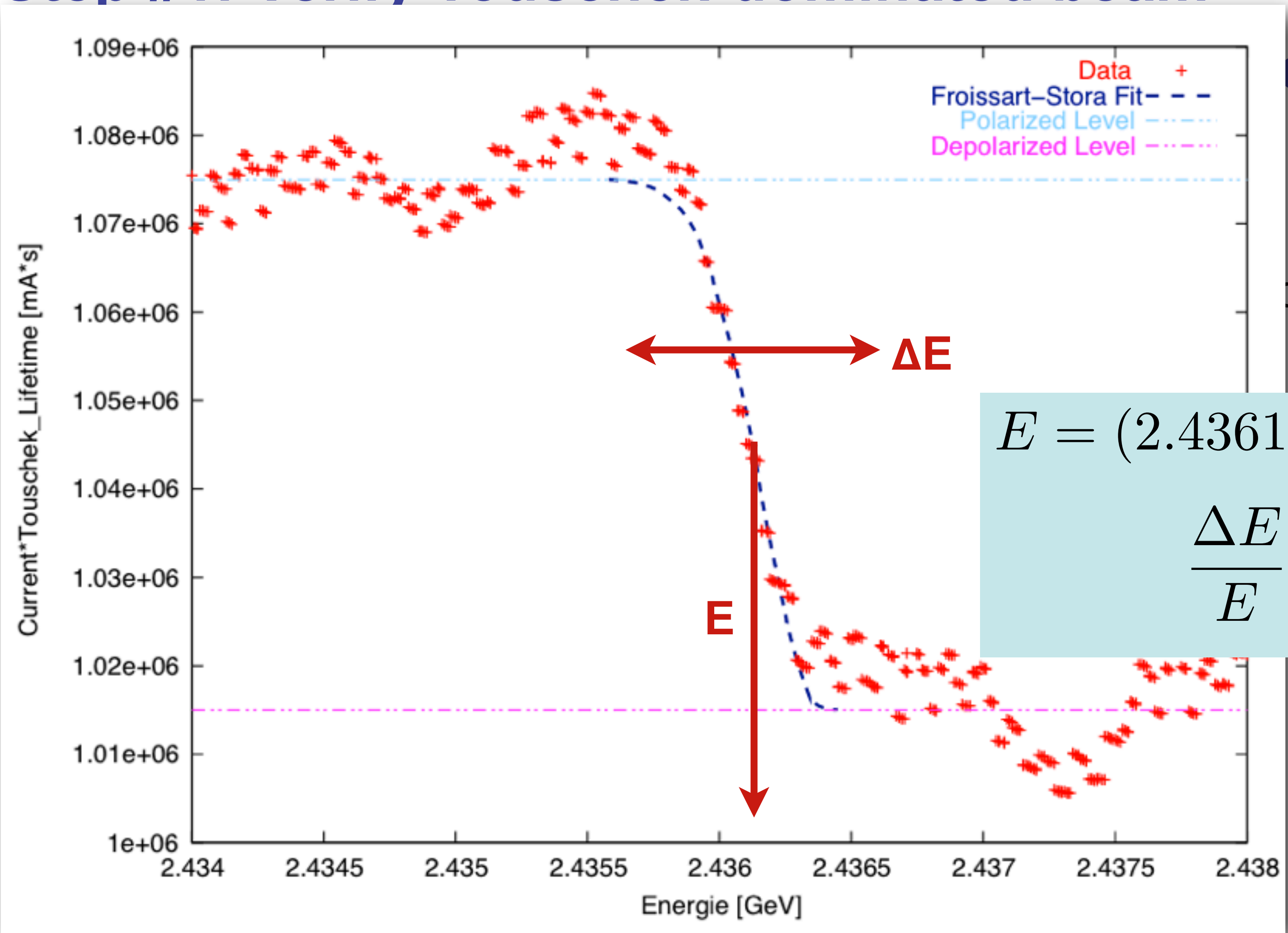

$$E = (2.4361 \pm 0.00018)\ \mathrm{GeV}$$

$$\frac{\Delta E}{E} = 7 \times 10^{-5}$$

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  - M … $= a\gamma$
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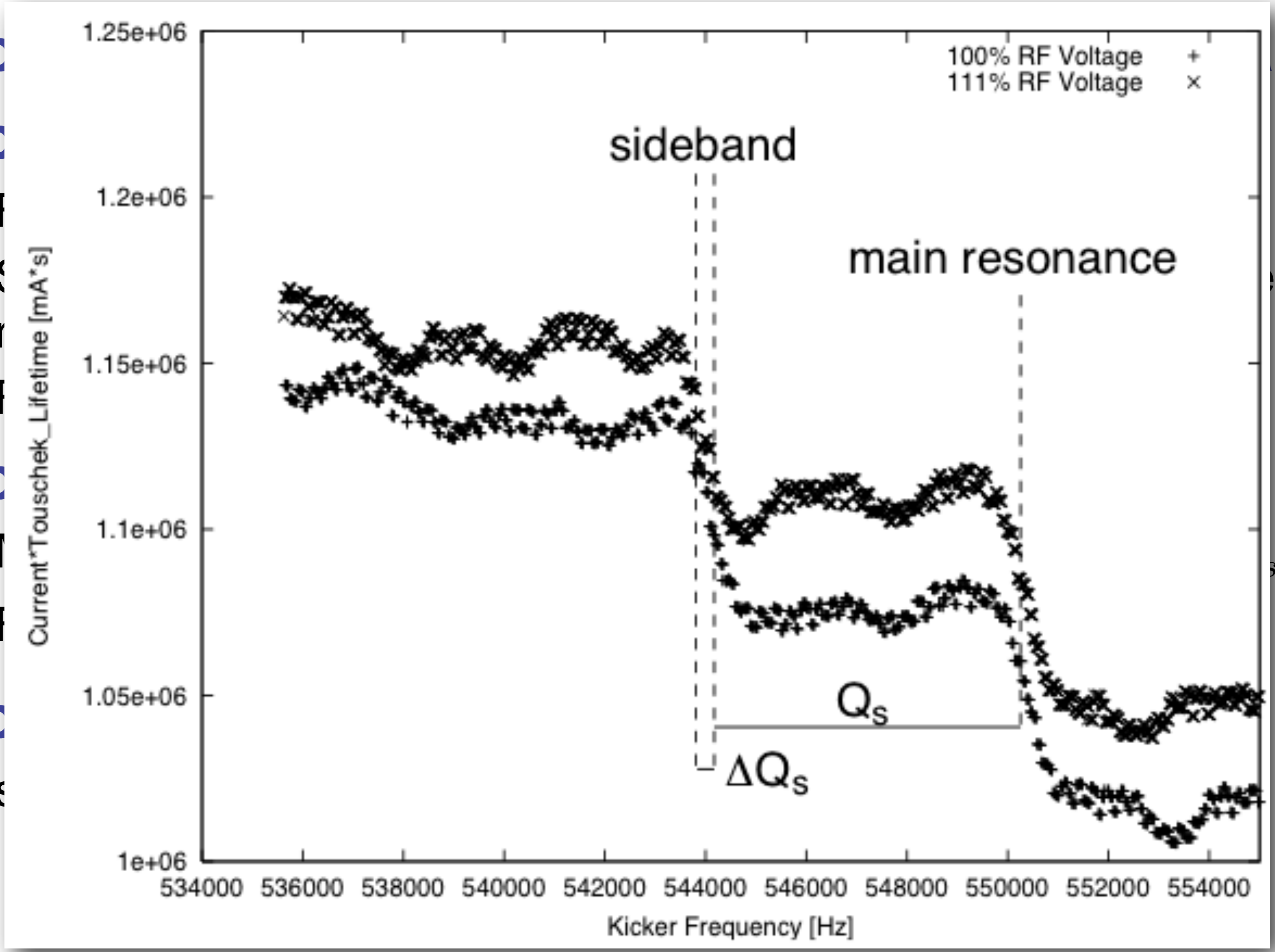
- Step
  - S



MAX-lab

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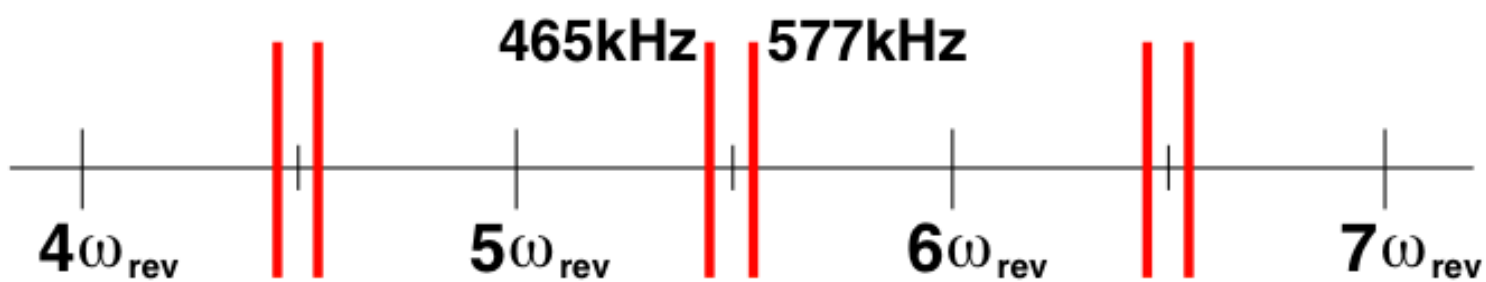
SLS: $E = 2.4$ GeV, $\omega_{rev} = 1042$ kHz

$\rightarrow \nu_{ST} = 5.4465$ corresponding to $\omega_{res-depol} \approx 465/577$ kHz



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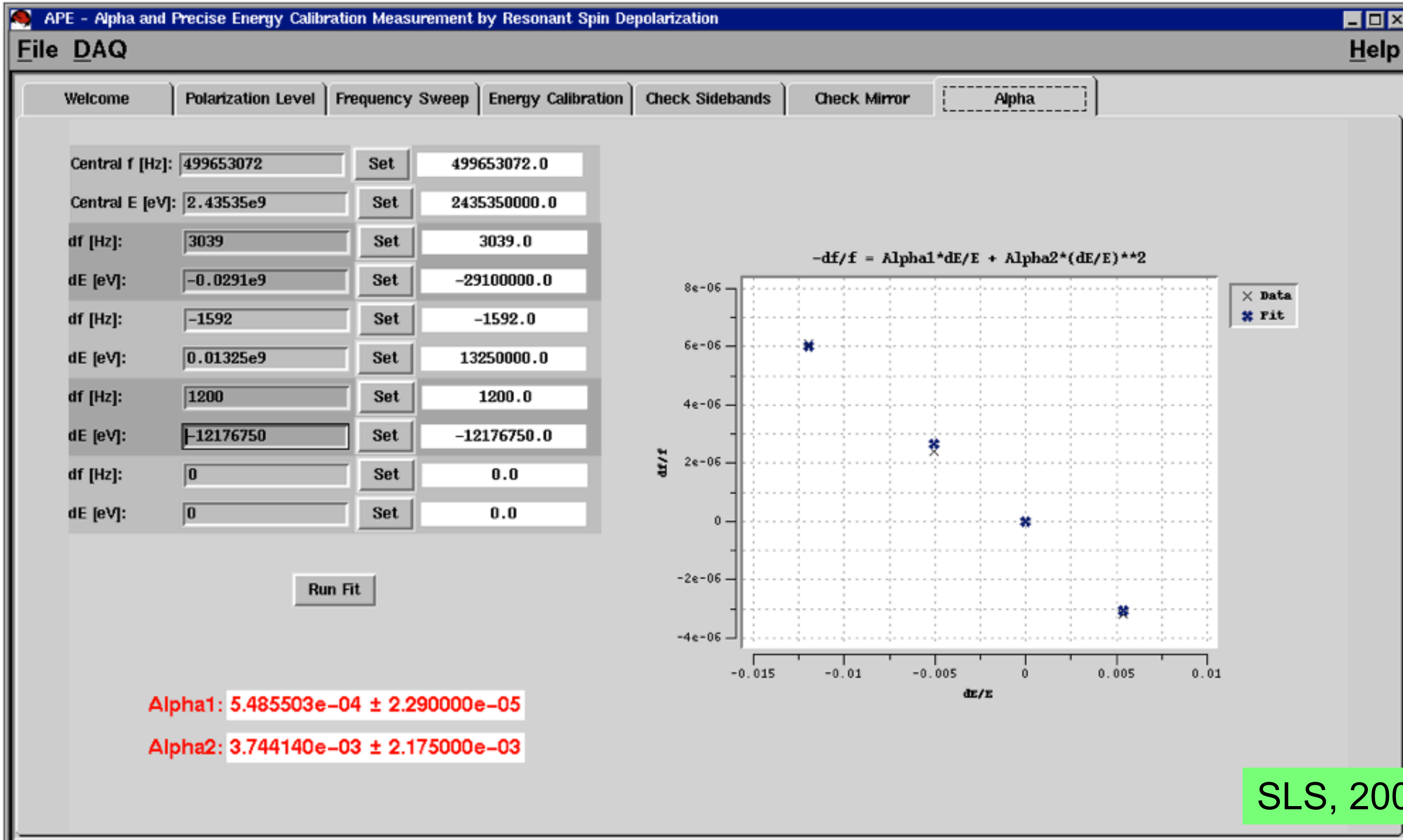

SLS, 2002

Example: RSD Measurement Campaign

- **Finally: 10^{-5} accuracy of energy calibration allows investigation of nonlinearity of momentum compaction**

- **This method (originally used in high-energy lepton rings) has by now been successfully applied at several light sources**
  - BESSY II
  - ALS
  - SLS
  - ANKA
  - DIAMOND
  - Australian Synchrotron
  - ...and most recently at SOLEIL