COUPLING AND BRIGHTNESS CONSIDERATIONS FOR THE MAX IV 3 GeV STORAGE RING

S.C. Leemann*, M. Eriksson, MAX IV Laboratory, Lund University, SE-22100 Lund, Sweden

Abstract

It is often suggested that the emittance coupling of a storage ring should be adjusted to the so-called diffraction limit corresponding to the shortest wavelength of interest. For 1 Å radiation this leads to a typical requirement of 8 pm rad vertical emittance. In ultralow-emittance storage rings like the MAX IV 3 GeV storage ring this corresponds to a comparably large setting of the emittance coupling (2.5%). This approach, however, does not produce the brightest radiation and needs to be revisited taking into account that overall photon brightness depends on the emittances of the electron beam and the intrinsic photon beam. This paper summarizes an analytic approach to maximizing brightness as a function of emittance coupling while retaining sufficient lifetime. Instead of "meeting the diffraction limit", we further reduce the coupling, thus increasing both the brightness and transverse coherence of the emitted radiation. We derive that reducing the MAX IV 3 GeV storage ring's vertical emittance to 2 pm rad (0.6% coupling) will increase brightness and transverse coherence by almost a factor two while 10 h overall lifetime can still be achieved.

INTRODUCTION

The MAX IV facility will use a 3 GeV linac and two storage rings to deliver synchrotron radiation to a broad and international user community across a wide spectral range and covering different temporal scales [1, 2]. The MAX IV 3 GeV storage ring has been optimized [3, 4] for insertion devices (ID's) producing high-brightness x-rays. A typical scenario is radiation at 1 Å produced in an in-vacuum undulator of roughly 4 m magnetic length. The intrinsic photon beam (i.e. the photon beam emitted by a single electron) emerging from such an ID [5] has a diffraction-limited RMS source size given by $\sigma_r = \sqrt{2L\lambda}/4\pi$ and its central cone has an RMS width $\sigma_{r'} = \sqrt{\lambda/2L}$ where L is the magnetic length of the ID and λ the (fundamental) emitted wavelength. This leads to the often quoted emittance criterion for diffraction limited radiation $\varepsilon_r = \sigma_r \sigma_{r'} = \lambda/4\pi$, which calls for 8 pm rad for 1 Å radiation.

In existing 3rd generation light sources, where the equilibrium emittance $\varepsilon_0 = \varepsilon_x + \varepsilon_y$ is several orders of magnitude larger than 8 pm rad, the conventional approach is to adjust the emittance coupling $\kappa = \varepsilon_y/\varepsilon_x$ by e.g. skew quadrupoles so that $\varepsilon_y \approx \varepsilon_r$. For small values of coupling $\varepsilon_x = \varepsilon_0/(1 + \kappa) \approx \mathrm{const.}$ The Touschek lifetime then scales like $\sqrt{\kappa}$ and the emittance coupling is therefore usually set "at the diffraction limit" and not much lower in order not to reduce overall beam lifetime.

This treatment, however, neglects the actual radiation process in the ID[1]. The brightness of the emitted radiation depends on the overlap of electron beam *and* intrinsic photon beam in phase space. This overlap, assuming a dispersion-free straight, is calculated as [5]

$$\Sigma_{x,y} = \sqrt{\sigma_r^2 + \sigma_{x,y}^2} = \sqrt{\frac{2L\lambda}{(4\pi)^2} + \varepsilon_{x,y}\beta_{x,y}}, \quad (1)$$

$$\Sigma_{x',y'} = \sqrt{\sigma_{r'}^2 + \sigma_{x',y'}^2} = \sqrt{\frac{\lambda}{2L} + \frac{\varepsilon_{x,y}}{\beta_{x,y}}}, \quad (2)$$

where $\beta_{x,y}$ is the beta function in the ID and we have assumed $\alpha_{x,y} = 0$ so that $\gamma_{x,y} = 1/\beta_{x,y}$. The spectral brightness of the emitted radiation is the flux at a specific wavelength[2] emitted from within a specific volume of transverse phase space volume, i.e.

$$\mathcal{B}(\lambda) = \frac{\mathcal{F}(\lambda)}{(2\pi)^2 \mathcal{E}_x \mathcal{E}_y},$$

where $\mathcal{E}_{x,y} = \Sigma_{x,y}\Sigma_{x',y'}$ describes the effective emittance, i.e. the emittance of the overlap between electron beam and intrinsic photon beam[3]. In a nutshell, this motivates the push for low-emittance rings running with large amounts of stored current and long undulators with short period lengths.

Finally, since the transversely coherent flux emitted at a specific wavelength λ is given by

$$\mathcal{F}_c(\lambda) = \mathcal{B}(\lambda)\left(\frac{\lambda}{2}\right)^2,$$

the coherent fraction of the emitted radiation can be expressed as

$$\begin{aligned}
f_c(\lambda) &= \frac{\mathcal{F}_c(\lambda)}{\mathcal{F}(\lambda)} = \frac{(\lambda/4\pi)^2}{\mathcal{E}_x \mathcal{E}_y} \\
&= \frac{1}{\sqrt{1 + (\frac{\sigma_x}{\sigma_r})^2}\sqrt{1 + (\frac{\sigma_{x'}}{\sigma_{r'}})^2}} \times \\
&\quad \frac{1}{\sqrt{1 + (\frac{\sigma_y}{\sigma_r})^2}\sqrt{1 + (\frac{\sigma_{y'}}{\sigma_{r'}})^2}}. \quad (3)
\end{aligned}$$

---

[1] This approach also entirely neglects the horizontal plane and tacitly accepts that ID radiation will, at best, be diffraction-limited in the vertical plane only. Diffraction-limited light sources, i.e. storage rings where the equilibrium emittance is so low that both transverse planes can be operated at or below the diffraction limit, present a solution to this problem and have recently become a topic of great interest [6, 7].

[2] This spectral flux $\mathcal{F}(\lambda)$ is conventionally defined as the number of photons emitted per second and per 0.1% BW ($\Delta\lambda/\lambda$). It is proportional to the stored current and the number of undulator periods.

[3] A noteworthy consequence is that intrinsic photon and electron emittances are added linearly, not quadratically.

---

* simon.leemann@maxlab.lu.se

It is interesting to note here, that even if the electron beam were perfectly matched to the intrinsic photon beam in transverse phase space, the coherent fraction achieved in this way would be only 25%. Increasing the coherent fraction beyond this level requires diminishing the electron beam's transverse phase space volume with respect that of the intrinsic photon beam.

MATCHING OPTICS TO ID RADIATION

Low-emittance storage rings need to be matched locally to strong ID's to prevent beta beating and tune shifts away from the working point [8]. In addition to such matching, the optics of the ring also need to be matched globally to a specific type of ID and a certain wavelength of interest in order to achieve highest brightness radiation. In 3rd generation light sources the source of radiation is a sheet beam where the horizontal emittance is much larger than that of the emitted radiation, while the vertical emittance and optics are usually the focus of matching efforts[4].

A first attempt at matching could be made by calling for perfect matching of the vertical beta function to a specific ID

$$\beta_y \overset{!}{=} \beta_r = \frac{\sigma_r}{\sigma_{r'}} = \frac{L}{2\pi}, \qquad (4)$$

which turns out to be independent of wavelength. This, however, leads to very small beta functions in the ID's which, in connection with the narrow gaps (4 mm are typically assumed), quickly leads to acceptance and hence lifetime issues[5]. On the other hand, one can easily show that $\beta_y = L/2$ ensures maximum acceptance for a given ID and gap setting. In the MAX IV 3 GeV storage ring, assuming a typical in-vacuum undulator of 4 m magnetic length, this has led to a choice of $\beta_y = 2$ m at the center of the long straight sections.

In terms of optics matching to the ID, this choice of β_y is a factor π too large and hence the overlap needs to be improved by appropriate choice of emittance coupling. Assuming the MAX IV 3 GeV storage ring is operated "at the diffraction limit" $\varepsilon_y = 8$ pm rad[6], inspection of Eqs. 1 and 2 reveals that for the typical in-vacuum undulator, in the vertical plane the angular overlap is dominated by the intrinsic photon beam, while the spatial mismatch is caused by the height of the electron beam in the ID. Figure 1 illustrates this situation: the horizontal mismatch between intrinsic photon beam and electron beam is very large while the vertical overlap can be considerably improved by choosing a reduced coupling.

---

[4]In addition, horizontal optics in straight sections are often also constrained by other requirements, e.g. large β_x required for injection.

[5]Note also, even if the vertical emittance is set at the diffraction limit and the vertical beta function is adjusted according to Eq. 4, the effective vertical emittance will still be twice the vertical intrinsic photon beam emittance according to Eqs. 1 and 2. This already indicates that a further reduction of vertical emittance pays off.

[6]Note this corresponds to $\kappa = 2.5\%$, a very relaxed requirement for emittance coupling compared to existing 3rd generation light sources. This is a consequence of the ultra-low emittance lattice of the MAX IV 3 GeV storage ring.
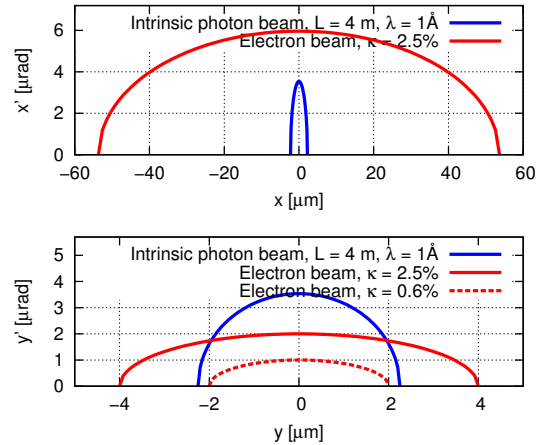
Figure 1: RMS beam size of the intrinsic photon and electron beam in phase space assuming 1 Å radiation emitted from a 4 m undulator installed in a long straight section of the MAX IV 3 GeV storage ring.

MAXIMIZING BRIGHTNESS AND COHERENCE

Since for very small values of coupling, $\mathcal{E}_x \approx$ const, as a consequence of $\varepsilon_x \approx \varepsilon_0 =$ const (in the absence of IBS), the brightness scales as

$$\mathcal{B}(\lambda) \propto \frac{1}{\mathcal{E}_y} = \frac{1}{\Sigma_y \Sigma_{y'}},$$

and hence one can expect a substantial brightness increase by reducing the vertical electron emittance. The results of an exact calculation assuming two typical ID's are displayed in Fig. 2.
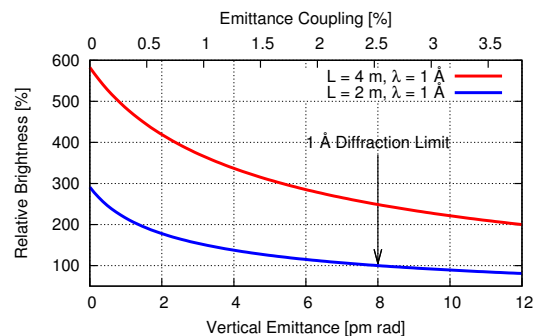

Figure 2: Brightness increase as emittance coupling is reduced for two ID's tuned to 1 Å. The brightness values have been normalized to that of the short ID operated at 8 pm rad. The effect of IBS has not been included.

Although 2 pm rad is a very low vertical emittance, it corresponds to 0.6% coupling which is comparable to presently operating light sources. Tracking studies for the MAX IV 3 GeV storage ring [4, 9] indicate roughly such a level of emittance coupling can be expected as a result of imperfections (magnet and alignment errors) as well as application of orbit correction and linear optics corrections.

Tracking studies have also quantified the decrease of Touschek lifetime and the emittance increase caused by IBS at 500 mA as a result of operating at reduced coupling. These studies have shown that the increase of IBS blow-up can be counteracted by bunch lengthening using Landau cavities. Furthermore, although Touschek lifetime reduces by roughly 50% as a consequence of the coupling reduction, with bunch lengthening from Landau cavities operated at the third harmonic (leading to stretching by roughly a factor five over the natural bunch length [10]) it should remain around 25 h, sufficient to ensure overall lifetime around 10 h.

Figure 3 shows the spectral brightness for a 3.3 m long in-vacuum undulator operated in the MAX IV 3 GeV storage ring at 500 mA. The roughly two-fold brightness in-
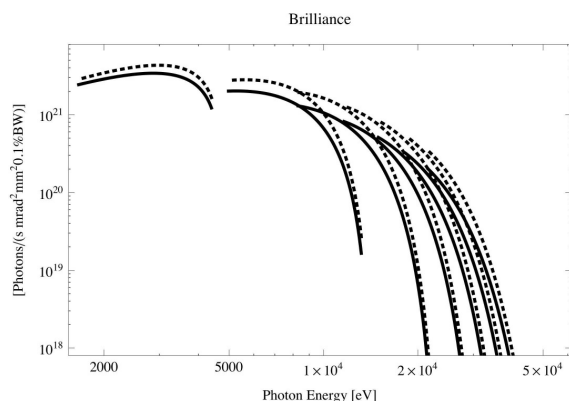

Figure 3: Spectral brightness at peak energy for a 3.3 m long in-vacuum undulator (18 mm period, 4 mm gap, 1.15 T effective field) installed in the MAX IV 3 GeV storage ring at 500 mA stored current for a vertical emittance of 8 pm rad (solid line) and 2 pm rad (dashed line) [7].

crease at wavelengths around 10 keV as a result of coupling reduction from the diffraction limit to 2 pm rad can be clearly recognized.

In addition to offering much higher brightness for x-rays, the reduced coupling also increases the transverse coherence of the emitted radiation. Figure 4 shows the degree of transverse coherence according to Eq. 3 for three undulators in the MAX IV 3 GeV storage ring tuned to three different wavelengths. Roughly a factor of two increase in coherence at 1 Å when going from the diffraction limit to 2 pm rad vertical emittance can be recognized. Note, however, that even for an ideal sheet beam no more than 2.1% transverse coherence can be achieved at 1 Å because of the mismatch in the horizontal plane (cf. Fig. 1, top). Higher levels of transverse coherence can be observed for softer radiation, albeit with a reduced dependence on emittance coupling since the overlap becomes dominated by the intrinsic photon beam's phase space volume.

CONCLUSIONS

We have demonstrated that setting the vertical emittance below the diffraction limit can lead to a substantial in-
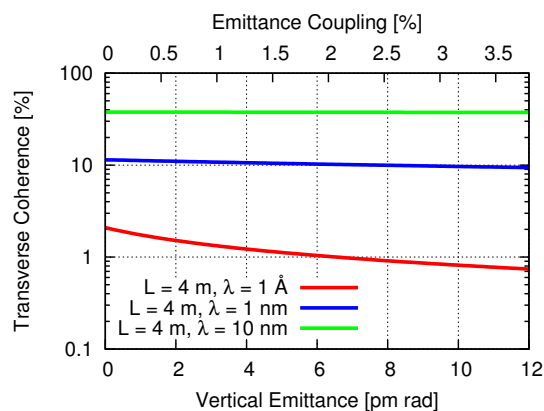

Figure 4: Degree of transverse coherence as a function of emittance coupling for radiation emitted at different wavelengths from ID's installed in a long straight section of the MAX IV 3 GeV storage ring.

crease of brightness for x-rays. The emittance coupling in the MAX IV 3 GeV storage ring is expected to be close to the required 0.6% and can be adjusted using the skew quadrupoles in the lattice. Since the impact of this reduced coupling on lifetime and IBS emittance blow-up can be mitigated by the Landau cavities, we will be able to offer a vertical emittance of 2 pm rad to our hard x-ray users. Transverse coherence is also improved by this reduction in coupling. However, the horizontal mismatch persists and hence transverse coherence at 1 Å remains low. A fully diffraction-limited storage ring delivering round beams to ID's below the diffraction limit in both planes simultaneously presents a possible solution.

REFERENCES

[1] MAX IV Detailed Design Report, available at `http://www.maxlab.lu.se/node/1136`.

[2] M. Eriksson et al., "The MAX IV Synchrotron Light Source", THPC058, IPAC 2011, San Sebastián, Spain.

[3] S.C. Leemann et al., Phys. Rev. ST Accel. Beams, **12**, 120701 (2009).

[4] S.C. Leemann, MAX-lab Internal Note 20121107, available at `http://www.maxlab.lu.se/node/999`.

[5] K.-J. Kim, Opt. Eng., Vol. **34**, No. 2, pp. 342-352 (1995).

[6] M. Borland, "Progress Towards Ultimate Storage Ring Light Sources", TUXB01, IPAC 2012, New Orleans, LA, USA.

[7] ICFA Beam Dynamics Newsletter, No. 57, April 2012, available at `http://icfa-usa.jlab.org/archive/newsletter/icfa_bd_nl_57.pdf`.

[8] E. Wallén, S.C. Leemann, "Strategy for Neutralizing the Impact of Insertion Devices on the MAX IV 3 GeV Storage Ring", TUP235, PAC'11, New York, NY, USA.

[9] S.C. Leemann, MAX-lab Internal Note 20130724, available at `http://www.maxlab.lu.se/node/999`.

[10] Å. Andersson et al., "The 100 MHz RF System for the MAX IV Storage Rings", MOPC051, IPAC 2011, San Sebastián, Spain.